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Paper: II

Group: B

Topic: Gauss's Theorem

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Gauss's Theorem

The flux through any closed surface is a measure of the total charge inside. On the other hand, a charge outside the surface will contribute nothing to the total flux. This is essence of Gauss's law.

Now, let us consider the total flux through a sphere of radius r which encloses a point charge

Sunday q at its centre.

Divide the sphere into small area elements, as shown in fig.

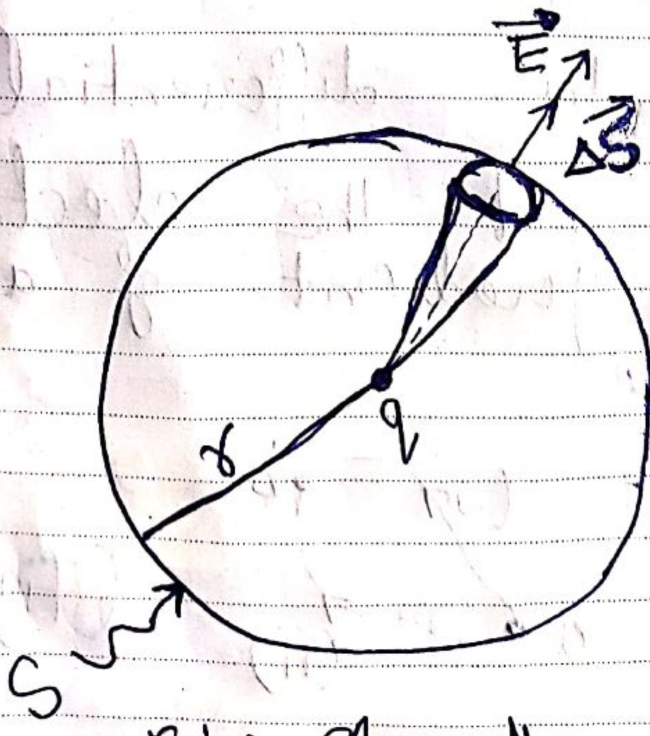

Fig: Flux through a sphere

The flux through an area element $\Delta\vec{S}$ is

$$\Delta\phi = \vec{E}\cdot\Delta\vec{S} = \frac{q}{4\pi\varepsilon_0 r^2}\,\hat{r}\cdot\Delta\vec{S} \quad \text{— (1)}$$

Since the normal to a sphere at every point is along the radius vector at that point, the area element $\Delta\vec{S}$ and $\hat{r}$ have the same direction.

Therefore,

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$$\Delta\phi = \frac{q}{4\pi\varepsilon_0 r^2}\,\Delta S \quad \text{— (2)}$$

Since the magnitude of a unit vector is 1.

The total flux through the sphere is obtained by adding up flux through all the different area elements:

$$\phi = \sum_{\text{all } \Delta S} \frac{q}{4\pi\varepsilon_0 r^2}\,\Delta S$$

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Since each area element of the sphere is at the same distance r from the charge,

$$\phi = \frac{q}{4\pi\varepsilon_0 r^2} \sum_{\text{all } \Delta S} \Delta S$$

$$\phi = \frac{q}{4\pi\varepsilon_0 r^2} \cdot S$$

Now S, the total area of the sphere, equals $4\pi r^2$. Thus,

$$\phi = \frac{q}{4\pi\varepsilon_0 r^2} \times 4\pi r^2$$

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$$\boxed{\phi = \frac{q}{\varepsilon_0}} \longleftarrow (3)$$

eq^n (3), illustrates the general result of electrostatics called Gauss's law.

Hence, Gauss's law states that Electric flux through a closed surface

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$$\Phi = \frac{q}{\varepsilon_0}$$

where, q = total charge enclosed by S.

This law implies that the total electric flux through a closed surface is zero if no charge is enclosed by the surface.

eq. (3) can be written as:

$$\boxed{\oint_S \vec{E} \cdot d\vec{a} = \frac{q}{\varepsilon_0}} \qquad \text{——— (4)}$$

where, q is enclosed by the surface

eq. (4) is an integral form of Gauss's law. We can readily turn it into a differential form.

Gauss's law in differential form

We know that,

$$\oint_S \vec{E} \cdot d\vec{a} = \frac{q}{\epsilon_0}$$

by applying the divergence theorem

$$\oint_S \vec{E} \cdot d\vec{a} = \int_V (\vec{\nabla} \cdot \vec{E}) \cdot d\tau$$

Rewriting q in terms of the charge density ρ, we have

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$$q = \int_V \rho \, d\tau$$

So Gauss's law becomes,

$$\int_V (\vec{\nabla} \cdot \vec{E}) \, d\tau = \int_V \left(\frac{\rho}{\epsilon_0} \right) d\tau$$

and since this holds for any Volume, the integerands must be equal:

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$$\boxed{\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0}}$$

It is Gauss's law in differential form.

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